

Article

Hybrid multiple structural break model for stock price trend prediction



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ABSTRACT

Because of the noises from the internal and external factors, the uncertainty increases in the financial market. The challenges of nonlinearities, volatility clusters, and multiple structural breaks which entail risk. Due to the risk, the prediction task becomes more complex. First, this work proposes a hybrid model to predict the one-day future price for the stocks; MSFT, Apple, Goldman Sachs and JP Morgan use the Markov switching model coupled with radial basis function network for prediction. Second, this paper forecasts the buy/sell trading strategy using the proposed hybrid method. Also, this paper explores the risk of investment decisions and the trading performance based on different value at risk (VaR) methods. Finally, by comparing the proposed model results with the pure linear and non-linear models, the prediction efficiency is evaluated. The experimental results indicate the investment risk, and the investment trading strategy provides a better accuracy with the best investment decision for the selected stocks.

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1. Introduction

Structural break, volatility modelling, and forecasting problems are considered as important tasks in the financial time series applications. This paper introduces a novel hybrid architecture called Markov switching Autoregressive model (4) combined with Bayesian Regularized Radial Basis Function Neural Network MSAR (4) (BR-RBFN) as a forecaster. Recently, researchers are attracted towards the pole of the stock price trend prediction which is embodied an increasing development in the model building process for evaluating the better investment stocks in the trading business. These models provide an investment trading insight to the investors who are forging the business deals in the trading market. Now, this profit maximization model demands an efficient market situation where investors are expected to yield a higher profit. However, if the market is efficient, it is not always easy to anticipate an investor who can yield a maximum profit than those who are holding the random stocks (Malkiel, 2003). So, it is vital to evaluate

the unit root test with the presence of multiple structural changes for efficient market hypothesis.

Naturally, the impulses (structural changes, conflicting behaviour, and political policy changes) from the internal and the external factors are predominantly changing the price direction. Thus, these deterministic changes and inherent noises make the prediction task even more complicated. The cornerstone of the prediction is that if the stock returns are mean revert, it will be possible to forecast the future trend using the past information data, as given in Malkiel and Fama (1970). There are a number of studies dealing with the efficient market hypothesis by examining the random walk analysis using different unit root tests for the emerging market (Gough and Malik, 2005; Lim and Brooks, 2011; Mishra et al., 2015; Borges, 2010). However, very few studies have allowed the structural changes in the unit root test for improving the prediction performance of the considered model. A study in the US on monthly natural gas consumption with the presence of GARCH unit root test and structural breaks has suggested that there is strong evidence of stationarity in the data, as mentioned by Mishra and Smyth (2014). Using macroeconomic time series data, a comparative analysis of structural break has been analyzed for improving the prediction performance (Bauwens et al., 2015).

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Based on the mathematical Bai-Perron technique, the housing price of the United States has been predicted by the presence of multiple structural breaks (Barari et al., 2014).

A number of studies have been analyzed for understanding the importance of parametric and non-parametric models, which are used in the application of time series forecasting. In finance, stock return prediction of Intraday S&P 500 index has been evaluated to improve the prediction accuracy as discussed in Matías and Reboredo (2012). This study suggests that the non-linear models surpassed the traditional linear models such as random walk on the basis of both statistical and economic criteria. In order to estimate and fine-tune the parameters of the non-linear STAR model, an evolutionary programming based on genetic algorithm has been utilized for forecasting the Eurozone peripheral stock market provided in Avdoulas et al. (2015). This study discusses the important implication of the market efficacy and predictability in terms of the investment trading strategies.

In recent years, Artificial Neural Networks play a major role in predicting the stock price trends (Ghiassi et al., 2005; Roh, 2007; Hamzaçebi et al., 2009; Patel et al., 2015). Oil price forecasts have been reviewed using the following price forecasting techniques such as ANN, genetic algorithm, support vector machine and hybrid system, which have been elaborated in Sehgal and Pandey (2015). Thereafter, a number of advanced mathematical techniques including fuzzy logic, genetic algorithm, and Bayesian probability have been exploited to improve the stock price trend prediction (Atsalakis and Valavanis, 2009; Chen et al., 2007; Chang, 2012; Ticknor, 2013; Sun et al., 2014; Ariyo et al., 2014). To perceive the plague of overfitting and overtraining of the neural network and to improve the network generalization, many hybridized studies have been developed for stock price trend prediction (Babu and Reddy, 2014; Adhikari and Agrawal, 2014). For predicting the electricity price, a hybrid ANN model has been proposed by Chaâbane (2014). The potential benefits and impact of accurate stock market prediction of Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX), Dow-Jones Industrial Average (DIA), S&P 500 and IBOVESPA Stock Index have been investigated by Chen and Chen (2015). Similarly, a combination of genetic algorithm with support vector machine has been modified to uncover the short-term trading efficiency of ASE 20 Index, which has been evaluated by Karathanasopoulos et al. (2015). Although there are many potential linear and non-linear stock price trend prediction models available in the academic literature, it is worthy of mention that no single model is best in all situations (Zhang, 2004).

First, this paper proposes a novel architecture in order to improve the strength of each non-linear model such as Markov switching model and radial basis function network model to predict the one-day future stock price for the four major stock returns. Then, this proposed model finds its predictive ability and improves the network generalization, as well as overcoming the plague of network overfitting and overtraining. Though there are a number of studies available on risk modelling, the single model reliability is still questionable. To the best of our knowledge, the combination of two non-parametric models has not yet contributed to the burgeoning literature for predicting the future stock returns.

Second, this study predicts the buy/sell trading days for the considered stock returns to improve the trading performance using the hybrid optimal cost function. Also, this paper investigates the investment decision peril based on the value at risk forecasts (VaR) using the parametric methods such as MA, EWMA, GARCH, historical simulation and expected shortfall. Finally, by comparing the proposed model results with the traditional Random walk linear model (RW) and the non-linear models such as Smooth transition autoregressive model (STAR), Multilayer Perceptron network (MLP), and Random forest network, the prediction efficiency is evaluated.

The rest of the paper is structured as follows. Section 2 describes the methodology of the proposed hybrid model; Sections 3 and 3.1 consider the data and the technical indicators and their importance; Section 4 discusses empirical analysis; Section 4.1 confers the results of the predictive hybrid model; finally, Section 5 summarizes the conclusions of this study.

2. Methodology

2.1. Markov switching model

A non-linear threshold Markov switching model has potentially been employed to estimate the multiple structural changes and to predict the one-day future diurnal stock return patterns. The effects of structural shocks are characterized over two-regimes in the parameter estimation. During the model building process, this model's flexibility allows the different types of behavioural patterns for the considered stock returns at different points in time. Hence, it helps to capture the multiple structural changes in the data and provides useful signal about the market activities.

Basically, Markov switching model by Hamilton (1996) implies that the probability distribution of an unobserved variable y_t lying in the interval $\{a, b\}$ depends on their state at the time $t - 1$ and not on any previous states.

Let us consider the unobserved state variable be S_t . Now the series of each considered stock data can be represented as follows:

$$y_t = \mu_0 + \mu S_t + (\sigma^2 + \theta) S_t^{1/2} * \varepsilon_t \quad (1)$$

$$\mu S_t = \mu_1 S_{1t} + \dots + \mu_M S_{Mt} \quad (2)$$

$$\sigma^2 S_t = \sigma_1^2 S_{1t} + \dots + \sigma_M^2 S_{Mt} \quad (3)$$

where $\varepsilon_t \sim N(0, 1)$. In state 1, the expected returns and the error variances are represented as (μ, σ^2) . Similarly, in state 2, they are $(\mu_0 + \mu, \sigma^2 + \theta)$. The unknown parameters $\mu_0, \mu, \sigma_0^2, \sigma^2, p_{11}, p_{22}$ can be estimated using maximum likelihood method. An easily computable technique called the Hamilton filter gives statistically significant values for the unknown parameters based on its means.

2.2. Bayesian regularized radial basis function network

In 1988, Broomhead and Lowe exploited the use of radial-basis functions in the design of the neural network. The radial basis function network is a three-layered architecture labelled as the input layer, a hidden layer, and the output layer (Broomhead and Lowe, 1988) as illustrated in Fig. 1. The input layer consists of 10 nodes of input neurons X_i . The hidden layer consists of the same number of evaluation units as the size of the training sample N ; each unit is mathematically described by a radial basis function F that has the following structure:

$$F(x) = \sum_{i=1}^N \omega_i \varphi(X - X_i) \quad (4)$$

where $\{\varphi(X - X_i) | i = 1, 2, \dots, N\}$ is a set of non-linear function known as radial-basis functions and $\|\cdot\|$ denotes a norm. The known data points $X_i \in \mathbb{R}^{m_0}$, $i = 1, 2, \dots, N$ are considered to be the centres of the radial-basis functions. Basically, high-dimensional space leads to the problem of the multivariable interpolation problem (Davis, 1963). Therefore, this interpolation problem can be stated as follows:

$$F(X_i) = v_i, \quad i = 1, 2, \dots, N \quad (5)$$

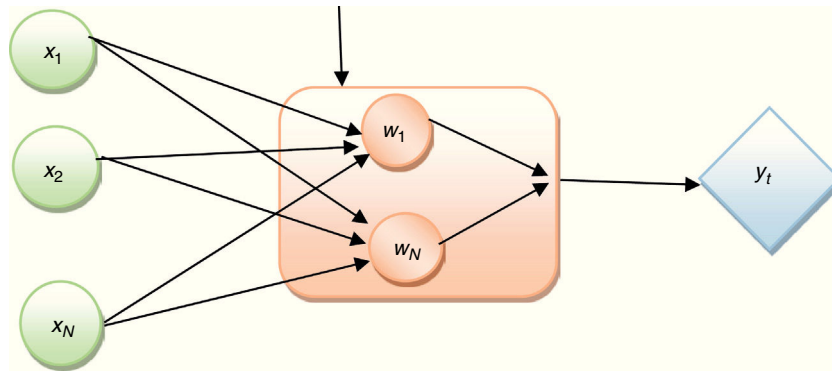


Fig. 1. The three-layered feed-forward radial basis function network.

Now, the simultaneous linear equation for the unknown coefficients of expansion $\{w_i\}$ are given by

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} & \dots & \varphi_{1N} \\ \varphi_{21} & \varphi_{22} & \dots & \varphi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_{N1} & \varphi_{N2} & \dots & \varphi_{NN} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} \quad (6)$$

Let $\mathbf{d} = [d_1, d_2, \dots, d_N]^T$; $\boldsymbol{\omega} = [\omega_1, \omega_2, \dots, \omega_N]^T$; $\boldsymbol{\phi} = \{\varphi_{ij}\}_{i,j=1}^N$ (7)

The $N \times 1$ vector \mathbf{d} and $\boldsymbol{\omega}$ represent the desired response vector and linear weight vector, respectively. $\boldsymbol{\phi}$ denotes a $N \times N$ matrix with the elements φ_{ij} .

The interpolation matrix is represented as

$$\boldsymbol{\phi}\boldsymbol{\omega} = \mathbf{X}; \quad (8)$$

The weight vector in the hidden layer can be solved by

$$\boldsymbol{\omega} = \boldsymbol{\phi}^{-1}\mathbf{X} \quad (9)$$

Henceforth, each computational unit in the hidden layer of the network that can be evaluated using the Gaussian function is defined by

$$\varphi_j(X) = \varphi(X - X_j) \quad (10)$$

$$\varphi_j(X) = \left(-\frac{1}{2\sigma_j^2}X - X_j^2 \right), \quad j = 1, 2, \dots, N \quad (11)$$

where σ_j is a measure of the width of the j th Gaussian function with the centre X_j .

The output layer consists of a single computational linear unit. Typically, the size of the output layer is much smaller than that of the hidden layer. For network training, the Bayesian regularized method has been utilized to estimate the optimum network parameters such as spread and weights. Thus, the optimized network's parameters provide better prediction accuracy and improve the network generalization by removing the plague of network overfitting and overtraining. This method adds the error term in the optimal cost function as follows:

$$y_t = uE_S + vE_w \quad (12)$$

where y_t is optimal cost function; E_S is sum of squared errors of the considered stock returns; E_w is sum of square of network weights; and u and v are the cost function parameters.

Using the Bayes' rule, the density function can be represented as follows:

$$P(w|u, v, S) = \frac{P(S|w, v, M)P(w|u, M)}{P(S|u, v, M)} \quad (13)$$

where w is network weight; S is considered stock data; and M is network model.

By solving Hessian matrix of y_t , the optimal function converges to its minimum point at $w^{minimum\ point}$. The regularized parameters u and v are evaluated using the Gauss-Newton approximation method given in [Foresee and Hagan \(1997\)](#).

2.3. Hybrid predictive method

In this study, two widely popular non-linear models, Markov switching AR (4) and Radial Basis Function network model, are exploited to predict the one-day future stock returns. First, this idea was proposed by [Zhang \(2004\)](#), and was further revised and explored by [Khashei and Bijari \(2011\)](#). From their studies, it is observed that the aggregate of two single models can efficiently improve the forecast accuracy of the considered stock return data than any single linear and nonlinear models do.

The proposed hybrid model is considered as a function of two non-linear components.

Thus,

$$y_t = f(N_{1t}), \quad i = 1, 2 \quad (14)$$

where y_t represents each stock return observations at time t . N_{1t} and N_{2t} are the non-linear components of Markov switching AR (4) model and radial basis function network model, respectively. Now,

$$y_t = N_{1t} + N_{2t} \quad (15)$$

This proposed method consists of the following three steps. First, the Markov switching AR (4) model has successfully captured the multiple structural changes and has effectively estimated the model parameters for the given stock returns. Such a model produces a series of predictions defined as N_{1t} . Hence, the obtained residual is denoted as e_t containing the non-linear patterns in the considered time series data, which can be represented as follows:

$$e_t = y_t - N_{1t} \quad (16)$$

$$e_t = g(e_{t-1}, e_{t-2}, \dots, e_{t-N}) + \varepsilon_t \quad (17)$$

where N_{1t} is the predicted values of the Markov switching model at time $1t$. ε_t is represented as the noise in the time series. Second, the radial basis function network model is utilized to explore the non-linear patterns of the obtained residuals, which are labelled as N_{2t} . Finally, the predictions of Markov switching AR (4) component (N_{1t}) and the radial basis function network component (N_{2t}) are recombined to generate an aggregate prediction.

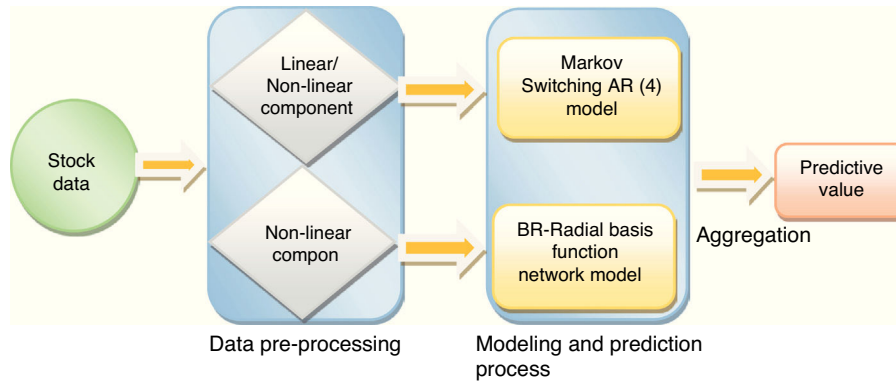


Fig. 2. Proposed hybrid model MSAR (4) BR-RBFN framework.

Thus, the estimated aggregate prediction is represented as below:

$$\hat{y}_t = \hat{N}_{1t} + \hat{N}_{2t} \tag{18}$$

Hence, this proposed hybrid method is not only improving the strength of single nonlinear model’s prediction accuracy, but also is expunging the issues of skewness and the serial correlation of the considered stock returns. Also, this method would overcome the plaque of network overfitting and overtraining using the probabilistic regularization method. This feature of the network randomly assigns weights to the network parameter, which is efficiently improving the network generalization. The proposed model framework is shown in Fig. 2.

3. Data

For empirical analysis, four major stocks, Microsoft (MSFT), Apple Inc. (Apple), Goldman Sachs (GMS) and JP Morgan Chase &Co (JPM), are selected to predict the one-day future stock price returns using the proposed hybrid model. The total number of samples for the considered stocks are 1257 daily trading days, from 3 January 2011 to 31 December 2015 including open, high, low, close price, trading volumes, and foremost seven technical indicators. From Fig. 1, it is shown that the four considered stock price returns exhibit different patterns in the upward direction, and provide the importance of trends to the investors or the traders so that they can make investment decisions based on the buy/sell trading strategy. The first 80% of the samples have been chosen as training set while the remaining 20% of the samples comprised of the testing set. In model building process, the training set is used to estimate the model parameters by increasing the number of hidden layer neurons until the parameters reached to its constant value and the testing set is used to select an appropriate model for forecasting the stock returns.

In this study, a comparative analysis is also conducted to test the network performance of the proposed model efficacy, with the pure traditional linear and some of the widely used nonlinear models such as Random walk, MSAR (4), STAR, MLP, BR-RBFN, and Random forest (Fig. 3).

3.1. Technical Indicators

Seven technical indicators were employed as a portion of the input variables to the network. It is often utilized by the investors who seem to predict the future price trend of an asset by looking at its past trend pattern. In the existing literature, it is vital to deduce the major indicators to make decisions in the investment trade market, as presented in Kara et al. (2011). These indicators are calculated using their formulae from the considered raw stock

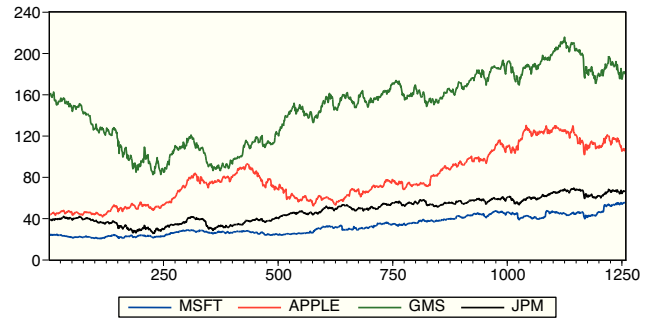


Fig. 3. Time series plot for all the considered stock returns.

data. Indicators and their respective formulae are summarized in Table 1.

4. Empirical analysis

4.1. Descriptive analysis

The summary statistics for daily considered stock returns from 2011 to 2015 is presented in Table 2. The daily mean for all the stock returns is small while the volatility of Apple, JPM and GMS are around 1.7% and the volatility of MSFT is around 1.5%. Further, the mean grows at a linear rate while volatility grows approximately at a square root rate. As seen over the time period, the expected return dominates the risk. The returns of Apple, JPM, and GMS have a very

Table 1 Selected technical indicators.

Name of indicators	Formula
EMA 5 and 10 days (exponential moving average)	$C_t - EMA(y)_{t-1} * \epsilon + EMA(y)_{t-1}$
Relative strength index	$100 - \frac{100}{1 + \left(\frac{\text{Average gain}}{\text{Average loss}} \right)}$
True strength index	$100 * \left(\frac{\sum_{k=1}^n C_{i-k}}{\sum_{k=1}^n C_{i-k}} \right) / n$
Average directional index	$SMA * \left(\frac{ +DI - (-DI) }{ +DI + (-DI) } \right)$
Williams R%	$\frac{HH_n - C_t}{HH_n - LL_n} * -100$
Stochastic K%	$\frac{C_t - LL_{t-n}}{HH_{t-n} - LL_{t-n}} * 100$
Stochastic D%	$\sum_{k=0}^n \frac{K_{t-k}}{n}$

C_t is the closing price, $\epsilon = 2/(1 + y)$, y is the time period for 5 or 10 days moving average, HH_n is the highest high, LL_n is the lowest low in past n days, $DI = +DM/TR$; $-DI = -DM/TR$, $+DM =$ Wilders exponential moving average of $+DM$; $-DM =$ Wilders exponential moving average of $-DM$; $TR =$ Wilders exponential moving average of TR .

Table 2
Descriptive statistics of the considered stock returns y_t .

Descriptive statistics	MSFT	APPLE	JPM	GMS
Mean	0.0655	0.0838	0.0579	0.0172
Std. Dev.	1.4831	1.6777	1.6975	1.6714
Maximum	10.4522	8.8741	8.4383	9.4705
Minimum	-11.3995	-12.3558	-9.4148	-10.095
Skewness	0.1235	-0.1751	-0.0484	-0.0657
Kurtosis	11.2837	7.6204	6.9716	6.7101
ARCH (12) LM test	22.48	34.89	18.34	28.05
Observations	1257	1257	1257	1257

small negative skewness, which implies that the probability of the occurrence of negative daily returns is higher on the considered stock returns with a high kurtosis. Finally, the ARCH-LM test shows that there is a significant time-varying volatility presence in the daily stock returns by rejecting the null hypothesis of no arch effect at 5 percent level for all the given stocks. For exploring the unit-root in the given series, the traditional unit root tests with and without trend have been performed using the ADF (Augmented Dickey-Fuller), the PP (Phillips-Perron) and KPSS (Kwiatkowski, Phillips, Schmidt, and Shin). The results are presented in Table 3. The stocks MSFT, Apple, GMS, and JPM confirm the presence of a unit root in the time series – meaning that in all the cases, the statistical values are fallen into the region of rejection at 1%, 5% and 10% significance level.

4.2. Predictive model results and discussion

The results for the multiple-break Bai and Perron (2003) unit root test are presented in Table 4. The multiple structural changes have been identified by Bai and Perron test statistics. It efficiently captures the maximum number of breaks $\hat{\nu}(TB)$ (maximum number of allowing breaks are Five) using the HAC estimator \hat{k}_T for the given stock returns. The function $\sup F_T(l+1|l)$ with 15% trimming value has been sequentially utilized to estimate the structural changes at 5% significance level. It is found that the stocks MSFT, Apple,

JPM, and GMS contain two structural breaks during the sequential process of multiple structural changes over the given time period. The major breaks have been identified in the years 2011, 2012, and 2015 over the given period. Therefore, the sequential test statistic $F_T(2|1)$, $F_T(3|2)$ for the given stocks is found to be (7.0392, 9.4654), (9.4654, 8.1842), (7.0702, 11.5746), and (10.1203, 7.6990). Similarly, for another test statistics UD_{Max} , WD_{Max} , values of MSFT, Apple, JPM, and GMS are (18.3191, 21.5508), (18.9309, 17.6915), (15.1614, 24.0520), and (20.2407, 20.2406), respectively. Finally, LWE criteria and Schwarz criteria significantly locate the breaks at the 5% level.

The multiple structural changes of the considered stock returns have been successfully modelled by Markov switching AR (4) model. The estimated parameters of Markov switching AR (4) model are presented in Table 5. The estimated returns (means) and risks (variances) for each stock data have been varied in the given two regimes. It is shown that the prices of MSFT and Apple stock returns are relatively higher than the stock returns of GMS and JPM. Also, each stock return is persistent for the shorter period than the longer time period. Naturally, the multiple structural changes cause the problem of poor prediction performance of the model. Thus, this Markov switching AR (4) model does not give any guarantee that it is the best single, reliable model for forecasting in every situation.

Next, the obtained residuals from the selected Markov switching AR (4) model is presented in Table 6. The residual's non-linear pattern has been identified by the BDS test, which suggests that the null hypothesis of i.i.d has fallen in the rejected region at 5% significance level, as given in Broock et al. (1996). Thus, there is clear evidence of the non-linear structure that exists in the residual data for the considered stock returns. To enhance the proposed model predictive performance, this non-linear residual structure is necessary to be fed into the neural network system to build a better model.

Table 7 shows the simulated results of the stocks MSFT, Apple, GMS, and JPM. The Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Root Mean Square Error (RMSE) are evaluated for the training, testing and the total data set to perceive

Table 3
Results of traditional unit root test.

Stocks	Intercept			Intercept and trend		
	ADF test	PP test	KPSS test	ADF test	PP test	KPSS test
MSFT	-0.2143	-0.0106	4.0529	-3.5384	-3.3787	0.3015
APPLE	-1.4165	-1.4143	3.3079	-1.8329	-1.8436	0.3042
JPM	-0.7311	-0.7447	3.7590	-2.8966	-2.8793	0.2837
GMS	-1.0705	-0.9924	3.0552	-2.9307	-2.8846	0.3764

Note: Significance at levels 5%.

Table 4
Results for multiple structural changes by Bai and Perron test (2003).

Stocks	No. of breaks	Sequential breaks		UDMax breaks	WDMax breaks	Schwarz breaks	LWZ breaks
		TB1	TB2				
MSFT	2	December 27, 2011 (7.0392)	November 06, 2012 (9.4654)	2 (18.3191)	2 (21.5508)	0 (-9.9623)	0 (-9.7945)
APPLE	2	April 22, 2014 (9.4654)	October 12, 2012 (8.1842)	1 (18.9309)	5 (17.6915)	0 (-9.7550)	0 (-9.5871)
JPM	2	July 19, 2012 (7.0702)	January 29, 2015 (11.5746)	3 (15.1614)	4 (24.0520)	0 (-9.6940)	0 (-9.5952)
GMS	2	July 20, 2012 (10.1203)	December 05, 2012 (7.6990)	1 (20.2407)	1 (20.2406)	0 (-9.7630)	0 (-9.5952)

Note: The values in brackets are value of structural break statistic compared with the Bai-Perron critical values (2003).

Table 5
The results of Markov switching AR (4) model.

Stocks	μ_0	μ	σ_0^2	σ^2	ρ_{11}	ρ_{22}
MSFT	0.0171	4.5762	0.0121	0.3421	0.9910	0.8750
Apple	0.1244	-4.2496	0.0402	0.5405	0.9826	0.8600
GMS	-0.9816	-0.3422	0.0885	0.0799	0.8891	0.9474
JPM	-0.2924	-0.5044	0.0965	0.0597	0.7983	0.9754

Note: (μ_0, μ) – estimated means of regime 1 and 2; (σ_0^2, σ^2) – estimated variances of regime 1 and 2; ρ_{11} – probability of being in regime one; ρ_{22} – probability of being in regime two.

Table 6
BDS test results on residuals from Markov switching AR (4) model.

Stocks	MSFT			Apple			JPM			GMS		
	0.3σ	0.5σ	0.7σ	0.3σ	0.5σ	0.7σ	0.3σ	0.5σ	0.7σ	0.3σ	0.5σ	0.7σ
$m = 2$	0.22	0.60	0.96	0.37	0.83	1.06	0.64	1.54	2.32	0.60	1.31	1.78
$m = 3$	0.19	0.88	1.80	0.27	1.03	1.78	0.65	2.46	4.82	0.59	2.16	3.86
$m = 4$	0.12	0.81	2.22	0.18	1.13	2.56	0.38	2.40	6.24	0.36	2.10	5.03
$m = 5$	0.61	0.69	2.43	0.80	0.91	2.79	0.20	2.03	6.96	0.19	1.80	5.70

Note: ε – the distance between the measure of standard deviations from the raw data; σ – standard deviations; m – embedding dimension.

the proposed model effectiveness of the given stock returns. With the lowest error measures, the proposed MSAR (4) BR-RBFN model can provide a good prediction value for the one-day future stock return, which is greater than 98.5% precision.

The simulated results of the proposed model for all the considered stocks are shown in Fig. 4(a)–(d). It is noticed that the actual values are not deviating much from its predicted value – meaning that the difference between the observed values and the predicted values are relatively small. Similarly, Fig. 5(a)–(d) shows the linear fitness curve of the stock returns MSFT and JPM. This study exploits a large number of data for testing the proposed model generalization with the presence of significant noises and structural changes. It is observed that the dotted line indicates the perfect fitness with slope one. The values spreading around the dotted line are the minimum error values of the actual and the predicted values. Thus, testing data provides the evidence of an appropriate model generalization and robustness with the reduced error values.

In order to explore the proposed model prediction accuracy, the error measures (MAE, MAPE, RMSE and proportion of the signs of return) are properly evaluated. The directional prediction changes for one-step ahead forecast are calculated using the DA test given in Pesaran and Timmermann (1992). As presented in Table 8, the goodness of fit for the pure linear and non-linear models is investigated by the statistical error values, based on one-step ahead forecasting values. The conclusions are twofold. First, the results of MSFT, APPLE, JPM, and GMS are heterogeneous based on the log-likelihood estimation, in terms of error measures and DA test measure. The MSAR (4) model tends to be the best nonlinear model in terms of MAE and RMSE error values. It is rarely outperformed by other non-linear models such as STAR, MLP, BR-RBFN and Random forest for one-day ahead prediction. As for MAPE

error values, the hybrid model results are outperforming the linear and other nonlinear models for all the considered stock returns at the near future period. Taking into account the proportion of Sign test, BR-RBFN model performs well than other models for the considered stock returns. The DA test of the null hypothesis of actual and predicted values is independent at 5% significance level for Random Walk, STAR, and MSAR (4) models that are rejected at the given time period. However, the MLP, BR-RBFN, Random forest, and Hybrid models do not reject the null hypothesis at any significance level. DA forecast accuracy is always providing a better indication of the investment trading strategy of the stock returns. Second, DM forecast accuracy test for RMSE examines whether the two models are statistically significant. Table 9 shows the forecast errors on e_1 and e_2 , which is shown in the upper triangular matrix. The lower triangular matrix is omitted because of the matrix symmetry. Now, the linear Random walk model is compared with all other non-linear models and vice versa. Then, the smaller RMSE values indicate the traditional linear model which naturally performs worse than other STAR, MSAR (4), BR-RBFN, Random forest and the proposed hybrid model. Despite all other nonlinear models being able to improve the forecast accuracy, the null hypothesis of true forecast errors for the considered stocks, which are identically zero, is not always constantly rejected. From the overall comparison, the results of nonlinear models show the hybrid MSAR (4) Br-RBFN model, which performs better than the linear and other nonlinear models such as STAR, MSAR (4), MLP, BR-RBFN, and Random forest in terms of the goodness of fit.

It is observed from Table 10 that the proposed MSAR (4) BR-RBFN model predictor performs significantly well than the other benchmark methods in terms of the information ratio. The proposed hybrid model surpasses the other non-linear models

Table 7
Forecast accuracy of hybrid Markov switching AR (4) BR-RBFN model.

Stock	MSFT			APPLE			JPM			GMS		
	MAE	MAPE	RMSE	MAE	MAPE	RMSE	MAE	MAPE	RMSE	MAE	MAPE	RMSE
Train set	0.0040	1.1415	0.0052	0.0035	2.7255	0.0046	0.0050	2.4829	0.0063	0.0053	1.3523	0.0069
Test set	0.0051	2.4941	0.0078	0.0079	1.4005	0.01042	0.0085	1.3737	0.0109	0.0064	2.0076	0.0083
Total set	0.0052	1.3134	0.0066	0.0040	1.8238	0.0053	0.0041	1.8533	0.0056	0.0043	1.0840	0.0054

Note: Significance level at 5%.

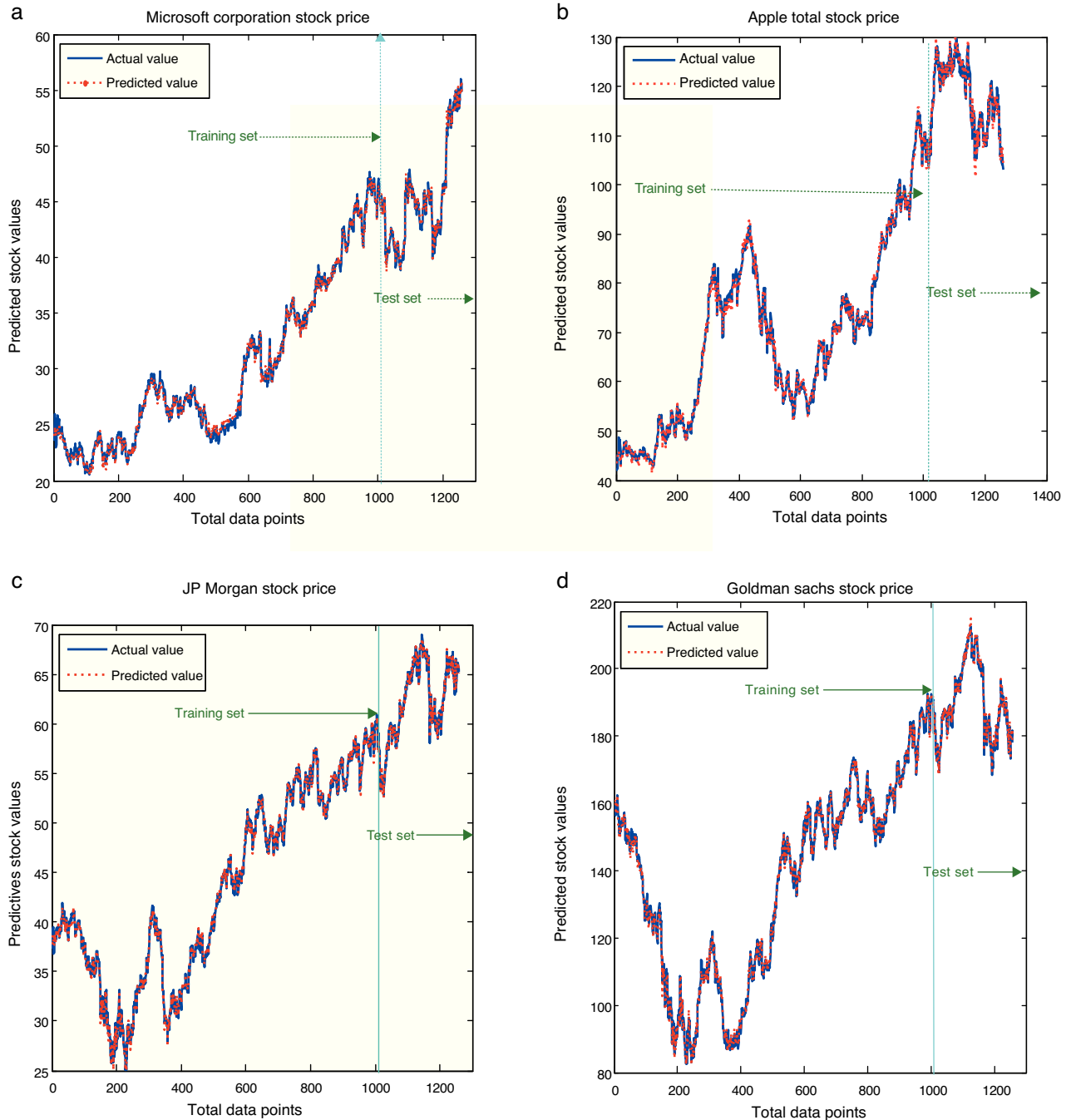


Fig. 4. (a)–(d) The predicting results of the MSFT, Apple, GMS, and JPM total data set.

in terms of the maximum drawdown return value. The hybrid model and BR-RBFN have quite high risk-adjusted returns than other linear and nonlinear models – meaning that the estimated expected returns of the selected stocks are engaged with zero risks.

In this study, the daily stock prices open, high, low, close and technical indicators are utilized to provide a better forecasting tool for the individual investors. The hybrid model could provide the best signal based on the evaluated risk values for the individual investors who want to invest in the market. The hybrid model MSAR (4) BR-RBFN and buy/sell trading performance are represented in Tables 11 and 12, respectively. The parametric methods such as

MA, EWMA, GARCH, historical simulation and expected shortfall are evaluated for the considered stock returns. The moving average (MA) model assumes that daily returns of the selected stocks get the same weights. The GMS provides higher risk value, approximately \$40.73 higher than other three assets. When exponentially weighted moving average (EWMA) improves, there is better volatility forecast by applying more weights to the nearest recent trading days. Thus, the volatility forecast of GMS is \$39.77. Generally, GARCH model not only allows the dynamic properties of volatilities but also promises to provide a better volatility forecast. The VaR of MSFT, Apple, GMS, and JPM are \$32.62, \$36.89, \$38.42, and \$42.35 respectively.

Table 8
Comparison of hybrid model forecast accuracy with the traditional linear and nonlinear models for the considered stocks.

MSFT	Random walk	STAR	MS AR (4)	MLP	BR-RBFN	Random forest	Hybrid
<i>MSFT</i>							
MAE	0.0098	0.5763	0.0044	0.0101	0.0109	0.0062	0.0052
MAPE	3.2431	1.1439	0.2566	7.5808	3.0202	2.3509	1.3134
RMSE	0.0146	0.6376	0.0051	0.0148	0.0165	0.0121	0.0066
DA test	-0.44	-0.61	-1.76	3.15	3.76	2.56	3.53
SIGN	0.1351	0.4356	0.2178	0.4526	0.5372	0.5145	0.8649
<i>APPLE</i>							
MAE	0.0108	1.5677	0.0069	0.0116	0.0132	0.0051	0.0040
MAPE	4.7271	1.8980	0.3439	5.3635	2.6008	3.0548	1.8238
RMSE	0.0151	1.6201	0.0071	0.0164	0.0179	0.0089	0.0053
DA test	-1.87	-0.67	-2.63	1.45	3.34	4.71	4.53
SIGN	0.3265	0.2643	0.3146	0.2967	0.3021	0.3521	0.3817
<i>JPM</i>							
MAE	0.0127	0.5173	0.0036	0.0326	0.0135	0.0044	0.0041
MAPE	4.9969	1.9816	0.1994	2.0354	2.9967	2.0036	1.8533
RMSE	0.0159	0.5258	0.0036	0.0442	0.0183	0.0075	0.0056
DA test	-2.47	-1.25	-0.48	3.53	3.57	2.49	3.51
SIGN	0.4025	0.4632	0.3821	0.3545	0.3854	0.4054	0.4188
<i>GMS</i>							
MAE	0.0125	1.6770	0.0051	0.0127	0.0138	0.0142	0.0043
MAPE	2.4272	1.0631	0.1804	2.2459	4.0188	4.5880	1.0840
RMSE	0.0164	1.6803	0.0052	0.0166	0.0183	0.0176	0.0054
DA test	-4.78	-1.78	-0.58	2.31	3.56	2.14	2.54
SIGN	0.4456	0.4520	0.4331	0.4374	0.4256	0.4478	0.4521

Note: Significance level at 5%. The bold values represent the significance of minimum error value of the proposed Hybrid model compared with the other linear and nonlinear models based on the error measures.

Table 9
Diebold and Mariano test for RMSE with $d_t = |e_{1t}| - |e_{2t}|$ for linear and non-linear model.

		e_1						
		RW	STAR	MSAR (4)	MLP	BR-RBFN	Random forest	Hybrid
e_2	RW	-	8.6772	9.0264	9.8547	10.0321	10.5078	11.8165
	STAR		-	8.8635	8.4331	9.5072	7.5348	9.6432
	MSAR (4)			-	-2.2193	-1.1444	0.0235	-3.0691
	MLP				-	0.1162	0.4026	1.7502
	BR-RBFN					-	-4.3220	0.1512
	Random forest						-	0.0321
	Hybrid							-

Note: e_1 – forecast error from model 1; e_2 – forecast error from model 2.

Similarly, the historical simulation (HS) method relies on its past history; the expected next trading day return depends upon the expected past returns of the given stocks. This HS method effectively captures the non-linear dependencies in the stock returns.

JPM and GMS stock returns have quite high VaR values as \$48.50 and \$46.29 respectively. The expected shortfall method is utilized to calculate the expected return for all the trading days of the selected stocks, which have more negative value than the VaR value of

Table 10
Buy/sell summaries of hybrid model trading performance.

	Random walk	STAR	MS AR (4)	MLP	BR-RBFN	Random forest	Hybrid
<i>MSFT</i>							
Information ratio	-0.15	0.57	1.62	1.75	1.93	1.76	2.07
Sharpe ratio	-0.27	0.11	0.37	1.23	1.56	1.64	2.01
Maximum draw down	-23.45	-20.13	-18.40	-16.67	-15.19	-14.84	-12.48
<i>Apple</i>							
Information ratio	-0.11	0.35	1.14	1.46	1.79	2.06	2.21
Sharpe ratio	-0.19	0.34	0.74	1.51	1.83	2.08	2.17
Maximum draw down	-21.67	-20.43	-19.61	-17.26	-15.84	-14.20	-13.08
<i>JPM</i>							
Information ratio	-0.29	0.14	0.42	1.39	1.86	2.13	2.64
Sharpe ratio	-0.20	0.13	0.48	1.87	2.08	2.27	2.52
Maximum draw down	-20.53	-14.78	-13.15	-13.89	-14.58	-12.06	-11.03
<i>GMS</i>							
Information ratio	-0.09	0.21	0.44	1.45	2.01	2.16	2.65
Sharpe ratio	-0.13	0.24	0.36	1.57	1.85	2.16	2.43
Maximum draw down	-16.84	-15.37	-15.89	-12.05	-11.01	-10.87	-9.04

Table 11
Forecasting risk value of hybrid model MSAR (4) BR-RBFN using the parametric method.

Stocks (US\$)	Time-dependent volatility			Historical simulation value (\$)	Expected short fall value (\$)
	Moving average (one-step ahead)	EWMA (\$)	GARCH (\$)		
MSFT	35.0432	32.5977	32.6208	36.6938	52.5434
Apple	37.2054	35.6068	36.8921	42.2945	59.6817
JPM	39.5432	34.5886	38.4210	48.5048	45.2442
GMS	40.7305	39.7677	42.3505	46.2918	58.5906

historical simulation. Basically, ES estimation needs a large number of samples to calculate the VaR value. The total number of samples for the selected stocks is 1257. Thus, the ES VaR value for MSFT, Apple, GMS and JPM are \$52.54, \$59.68 \$58.59 and \$45.24, respectively.

From Table 12, it is observed that the trading decision buy/sell points for the stocks MSFT, Apple, JPM and GMS have been forecasted using the hybrid model MSAR (4) BR-RBFN. For this analysis, the window size is decided at 5, based on the analysis that the potential stocks could easily be identified for the investment.

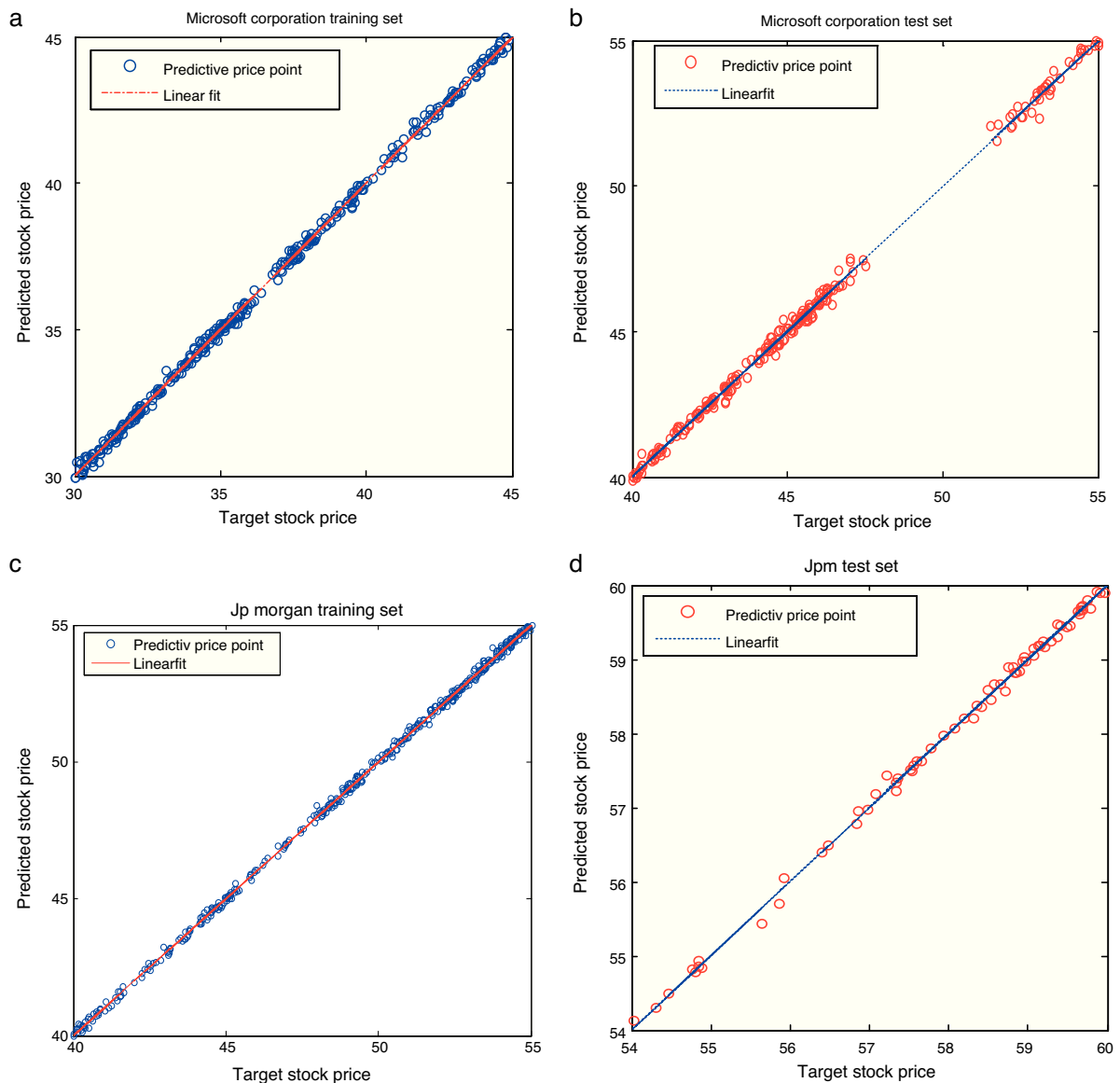


Fig. 5. Comparisons of MSFT and GMS target and predicted stock price values.

Table 12
Forecasting buy/sell points using hybrid model MSAR (4) BR-RBFN.

Buy/sell date	Buying price	Selling price
<i>MSFT</i>		
12/1/2015	25.47286693	34.52713
12/2/2015	25.58446788	34.41553
12/3/2015	26.56696714	33.43303
12/4/2015	24.93509126	35.06491
12/7/2015	25.03436098	34.96564
<i>Apple</i>		
12/1/2015	36.02798494	96.02798494
12/2/2015	35.28088117	95.28088117
12/3/2015	34.07620223	94.07620223
12/4/2015	37.56933743	97.56933743
12/7/2015	36.87427803	96.87427803
<i>JPM</i>		
12/1/2015	13.34713171	46.65286829
12/2/2015	14.16014059	45.83985941
12/3/2015	14.96505057	45.03494943
12/4/2015	13.10641082	46.89358918
12/7/2015	14.06635508	45.93364492
<i>GMS</i>		
12/1/2015	111.3986914	171.3986914
12/2/2015	108.6880585	168.6880585
12/3/2015	103.6173696	163.6173696
12/4/2015	108.4267622	168.4267622
12/7/2015	103.9791637	163.9791637

5. Conclusions

The findings showed that the proposed hybrid model MSAR (4) BR-RBFN is significantly improving the one-step-ahead predictive performance for the stock returns MSFT, Apple, GMS, and JPM. Additionally, this hybrid approach overcomes the limitations of single model predictability of the considered stock returns. Also, with the minimum error measures, this hybrid model MSAR (4)-BRRBFN can predict the future stock investment decisions based on the buy/sell trading signal for the given stock returns in the trading market. It is noteworthy to mention that the proposed MSAR (4)-BRRBFN predictor inherits its network generalization and overcomes the plague of network overfitting and overtraining.

Moreover, the experimental results of value at risk explores that the evaluation of investment risk and the investment trading strategy, providing a better investment risk accuracy with the lowest risk errors using the parametric methods such as MA, EWMA, GARCH and historical simulation in favour of the considered stock returns. The comparative results embody that the predictive efficacy of the proposed hybrid model significantly surpassed its pure linear and non-linear models. Thus, this model proves its effective predictive power and its reliability in terms of capturing the non-linear patterns with minimum error values. Thus, this predictive tool provides an information signal to the individual investors who are willing to forge in the market. So, this novel architecture highlights the importance of hybrid modelling and also provides a good financing tool for the individual investors.

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References

- Adhikari, R., Agrawal, R.K., 2014. A combination of artificial neural network and random walk models for financial time series forecasting. *Neural Comput. Appl.* 24 (6), 1441–1449.
- Ariyo, A.A., Adewumi, A.O., Ayo, C.K., 2014. March. Stock price prediction using the ARIMA model. In: 2014 UK Sim AMSS 16th International Conference on Computer Modelling and Simulation (UKSim). IEEE, pp. 106–112. <http://dx.doi.org/10.1109/UKSim.2014.67>.
- Atsalakis, G.S., Valavanis, K.P., 2009. Forecasting stock market short-term trends using a neuro-fuzzy based methodology. *Expert Syst. Appl.* 36 (7), 10696–10707.
- Avdoulas, C., Bekiros, S., Boubaker, S., 2015. Evolutionary-based return forecasting with nonlinear STAR models: evidence from the Eurozone peripheral stock markets. *Ann. Oper. Res.*, 1–27. <http://dx.doi.org/10.1007/s10479-015-2078-z>.
- Babu, C.N., Reddy, B.E., 2014. A moving-average filter based hybrid ARIMA-ANN model for forecasting time series data. *Appl. Soft Comput.* 23, 27–38.
- Bai, J., Perron, P., 2003. Computation and analysis of multiple structural change models. *J. Appl. Econ.* 18 (1), 1–22.
- Bauwens, L., Koop, G., Korobilis, D., Rombouts, J.V., 2015. The contribution of structural break models to forecasting macroeconomic series. *J. Appl. Econ.* 30 (4), 596–620.
- Borges, M.R., 2010. Efficient market hypothesis in European stock markets. *Eur. J. Finance* 16 (7), 711–726.
- Broock, W.A., Scheinkman, J.A., Dechert, W.D., LeBaron, B., 1996. A test for independence based on the correlation dimension. *Econom. Rev.* 15 (3), 197–235.
- Broomhead, D.S., Lowe, D., 1988. Radial basis functions, multi-variable functional interpolation and adaptive networks (No. RSRE-MEMO-4148). Royal Signals and Radar Establishment Malvern (United Kingdom).
- Chaâbane, N., 2014. A hybrid ARFIMA and neural network model for electricity price prediction. *Int. J. Electr. Power Energy Syst.* 55, 187–194.
- Chang, P.C., 2012. A novel model by evolving partially connected neural network for stock price trend forecasting. *Expert Syst. Appl.* 39 (1), 611–620.
- Chen, M.Y., Chen, B.T., 2015. A hybrid fuzzy time series model based on granular computing for stock price forecasting. *Inf. Sci.* 294, 227–241.
- Chen, Y., Yang, B., Abraham, A., 2007. Flexible neural trees ensemble for stock index modeling. *Neurocomputing* 70 (4), 697–703.
- Foresee, F.D., Hagan, M.T., 1997. June. Gauss-Newton approximation to Bayesian learning. In: International Conference on Neural Networks, vol. 3. IEEE, pp. 1930–1935.
- Ghiassi, M., Saidane, H., Zimbra, D.K., 2005. A dynamic artificial neural network model for forecasting time series events. *Int. J. Forecast.* 21 (2), 341–362.
- Gough, O., Malik, A., 2005. Random walk in emerging markets: a case study of the Karachi stock exchange. In: *Risk Management in Emerging Markets*. Palgrave Macmillan, UK, pp. 57–70.
- Hamilton, J.D., 1996. Specification testing in Markov-switching time-series models. *J. Econom.* 70 (1), 127–157.
- Hamzaçebi, C., Akay, D., Kutay, F., 2009. Comparison of direct and iterative artificial neural network forecast approaches in multi-periodic time series forecasting. *Expert Syst. Appl.* 36 (2), 3839–3844.
- Kara, Y., Boyacıoğlu, M.A., Baykan, Ö.K., 2011. Predicting direction of stock price index movement using artificial neural networks and support vector machines: the sample of the Istanbul stock exchange. *Expert Syst. Appl.* 38 (5), 5311–5319.
- Karathanasopoulos, A., Theofilatos, K.A., Sermpinis, G., Dunis, C., Mitra, S., Stasinakis, C., 2015. Stock market prediction using evolutionary support vector machines: an application to the ASE20 index. *Eur. J. Finance*, 1–19.
- Khashei, M., Bijari, M., 2011. A novel hybridization of artificial neural networks and ARIMA models for time series forecasting. *Appl. Soft Comput.* 11 (2), 2664–2675.
- Barari, M., Sarkar, N., Kundu, S., Chowdhury, K.B., 2014. Forecasting house prices in the United States with multiple structural breaks. *Int. Econom. Rev.* 6 (1), 1–23.
- Lim, K.P., Brooks, R., 2011. The evolution of stock market efficiency over time: a survey of the empirical literature. *J. Econ. Surv.* 25 (1), 69–108.
- Malkiel, B.G., 2003. The efficient market hypothesis and its critics. *J. Econ. Perspect.* 17 (1), 59–82.
- Malkiel, B.G., Fama, E.F., 1970. Efficient capital markets: a review of theory and empirical work. *J. Finance* 25 (2), 383–417.
- Matias, J.M., Reboredo, J.C., 2012. Forecasting performance of nonlinear models for intraday stock returns. *J. Forecast.* 31 (2), 172–188.
- Mishra, A., Mishra, V., Smyth, R., 2015. The random-walk hypothesis on the Indian stock market. *Emerg. Mark. Finance Trade* 51 (5), 879–892.
- Mishra, V., Smyth, R., 2014. Is monthly US natural gas consumption stationary? New evidence from a GARCH unit root test with structural breaks. *Energy Policy* 69, 258–262.
- Davis, P.J., 1963. *Interpolation and Approximation*, 2nd ed. Blaisdell, Waltham, MA, Dover, New York, 1975.
- Patel, J., Shah, S., Thakkar, P., Kotecha, K., 2015. Predicting stock and stock price index movement using trend deterministic data preparation and machine learning techniques. *Expert Syst. Appl.* 42 (1), 259–268.

- Pesaran, M.H., Timmermann, A., 1992. [A simple nonparametric test of predictive performance](#). *J. Bus. Econ. Stat.* 10 (4), 461–465.
- Roh, T.H., 2007. [Forecasting the volatility of stock price index](#). *Expert Syst. Appl.* 33 (4), 916–922.
- Sehgal, N., Pandey, K.K., 2015. [Artificial intelligence methods for oil price forecasting: a review and evaluation](#). *Energy Syst.* 6 (4), 479–506.
- Sun, Q., Che, W.G., Wang, H.L., 2014. Bayesian regularization BP neural network model for the stock price prediction. In: *Foundations and Applications of Intelligent Systems*. Springer Berlin, Heidelberg, pp. 521–531, http://dx.doi.org/10.1007/978-3-642-37829-4_45.
- Ticknor, J.L., 2013. [A Bayesian regularized artificial neural network for stock market forecasting](#). *Expert Syst. Appl.* 40 (14), 5501–5506.
- Zhang, G.P., 2004. A combined ARIMA and neural network approach for time series forecasting. *Neural networks*. IRM, Hershey, pp. 213–222, <http://dx.doi.org/10.4018/978-1-59140-176-6.ch011>.